Fall 2024 MATH 33A Worksheets - Week 1

September 2024

Problem 1. *Use Gauss-Jordan Elimination to solve the following systems of linear equations.*

(a)

$$\begin{aligned} x_1 - 4x_2 - x_3 &= 3 \\ 2x_1 - 8x_2 + x_3 &= 9 \end{aligned}$$

(b)

$$\begin{aligned} x_1 + 2x_2 - x_3 &= -1 \\ 2x_1 + 2x_2 + x_3 &= 1 \\ 3x_1 + 5x_2 - 2x_3 &= -1 \end{aligned}$$

Problem 2. *For each of the matrices below, decide if the matrix is in Reduced Row Echelon Form. Transform it to Reduced Row Echelon Form if it is not. Then, say whether the corresponding system of equations has 0 solutions, one solution, or infinitely-many solutions.*

(a)

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

(b)

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right]$$

(c)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Problem 3 (Textbook 1.1.17)**.** *Find solutions of the linear system*

$$\left| \begin{array}{l} x + 2y = a \\ 3x + 5y = b \end{array} \right|$$

Problem 4. *A real* ***cubic polynomial*** *is a function of the form* $p(x) = ax^3 + bx^2+cx+d$ *for real numbers* a, b, c, d. *Suppose we want to find a cubic polynomial* p *such that*

$$p(-1) = 5 \quad p(0) = 1 \quad p(1) = 1 \quad p(2) = 11$$

Turn these conditions into a system of linear equations and solve it to find the values of a, b, c, d.

Problem 5 (Textbook 1.2.22)**.** *We say two* $n \times m$ *matrices in Reduced Row Echelon Form are "of the same type" if they contain the same number of leading ones* ***and*** *those leading ones occur in the same position. For example, the following two matrices are of the same type:*

$$\begin{bmatrix} \textcircled{1} & 2 & 0 \\ 0 & 0 & \textcircled{1} \end{bmatrix} \text{ and } \begin{bmatrix} \textcircled{1} & 3 & 0 \\ 0 & 0 & \textcircled{1} \end{bmatrix}$$

How many different types of 2×2 *matrices are there?*

Problem 6. *The* dot products *of two vectors*

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ and } \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

in $\mathbb{R}^n$ *is defined by*

$$\mathbf{x} \cdot \mathbf{y} = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n.$$

We say that the vectors $\mathbf{x}$ *and* $\mathbf{y}$ *are* perpendicular *if* $\mathbf{x} \cdot \mathbf{y} = 0$. *Find all vectors in* $\mathbb{R}^3$ *that are perpendicular to the vectors*

$$\begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}.$$