# Fall 2024 MATH33A Worksheet 2: Sections 1.3, 2.1, 2.2

**Problem 1.** Find the rank of the following matrices. *(Hint: first find the Reduced Row-Echelon Form.)*

(a) $\begin{bmatrix} 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 4 \\ 3 & 2 & 4 \\ 0 & 5 & -5 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -3 \\ -6 & 9 \end{bmatrix}$

**Problem 2.** Compute the following or state that it is undefined. *(Compare the answers for (e) and (f).)*

(a) $\begin{bmatrix} 6 & 5 \\ 0 & -5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 10 \\ -4 & -3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ 5 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ (f) $x \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + y \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + z \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

**Problem 3.**

(a) Is $\begin{bmatrix} 3 \\ -8 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$?

(b) Is $\begin{bmatrix} -4 \\ -1 \\ -15 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 9 \\ 11 \end{bmatrix}$?

(c) Is $\begin{bmatrix} 5 \\ 2 \\ 7 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$?

**Problem 4.** Find the matrix of the following linear transformations.

(a) The transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ given by

$$\begin{aligned} y_1 &= 8x_1 + x_2 \\ y_2 &= 4x_1 - x_2 \\ y_3 &= -3x_1 + x_2 \end{aligned}$$

(b) The transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ given by

$$\begin{aligned} y_1 &= 3x_1 + 4x_2 + 5x_3 \\ y_2 &= -x_1 - x_2 - 5x_3 \end{aligned}$$

(c) Rotation clockwise by $90^\circ$ from $\mathbb{R}^2$ to $\mathbb{R}^2$

**Problem 5.** Find the inverse of the following matrices, or show there is no inverse.

(a) $\begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -4 \\ -2 & 8 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$

**Problem 6.** Describe what the following linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$ do geometrically. *(Hint: consider what the transformation does to* $\vec{e_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ *and* $\vec{e_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$*.)*

(a) $T(\vec{x}) = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \vec{x}$ (b) $f(\vec{x}) = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \vec{x}$ (c) $R(\vec{x}) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \vec{x}$