Fall 2024 MATH33A Worksheet 4: Textbook 3.2, 3.3

Problem 1. For each of the following matrices, find a basis for the kernel and find a basis for the image.

(a) $\begin{bmatrix} 1 & 7 \\ -1 & -6 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 5 \\ 3 & 9 & 1 \\ -2 & -6 & 0 \end{bmatrix}$

Problem 2. For each of the following sets, decide whether it is a subspace of $\mathbb{R}^3$. Justify your answer.

(a) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x^2 + y^2 + z^2 = 0 \right\}$

(b) $\left\{ \begin{bmatrix} 2s+1 \\ t-s \\ t \end{bmatrix} : s, t \in \mathbb{R} \right\}$

(c) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : 2x + 3y - z = 0 \in \mathbb{R} \right\}$

Problem 3. Determine whether the given vectors are linearly dependent or linearly independent.

(a) $\begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 0 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 3 \\ 1 \end{bmatrix}$

Problem 4. For which values of k is the following a basis of $\mathbb{R}^4$:

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 14 \\ k+6 \\ 24 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -1 \\ k^2 - 12 \end{bmatrix}$$

Problem 5. Find a basis for the subspace of $\mathbb{R}^4$ defined by

$$x_1 - 2x_2 + 4x_3 - 4x_4 = 0.$$